

Calculation Times in Stable32

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- **Introduction**

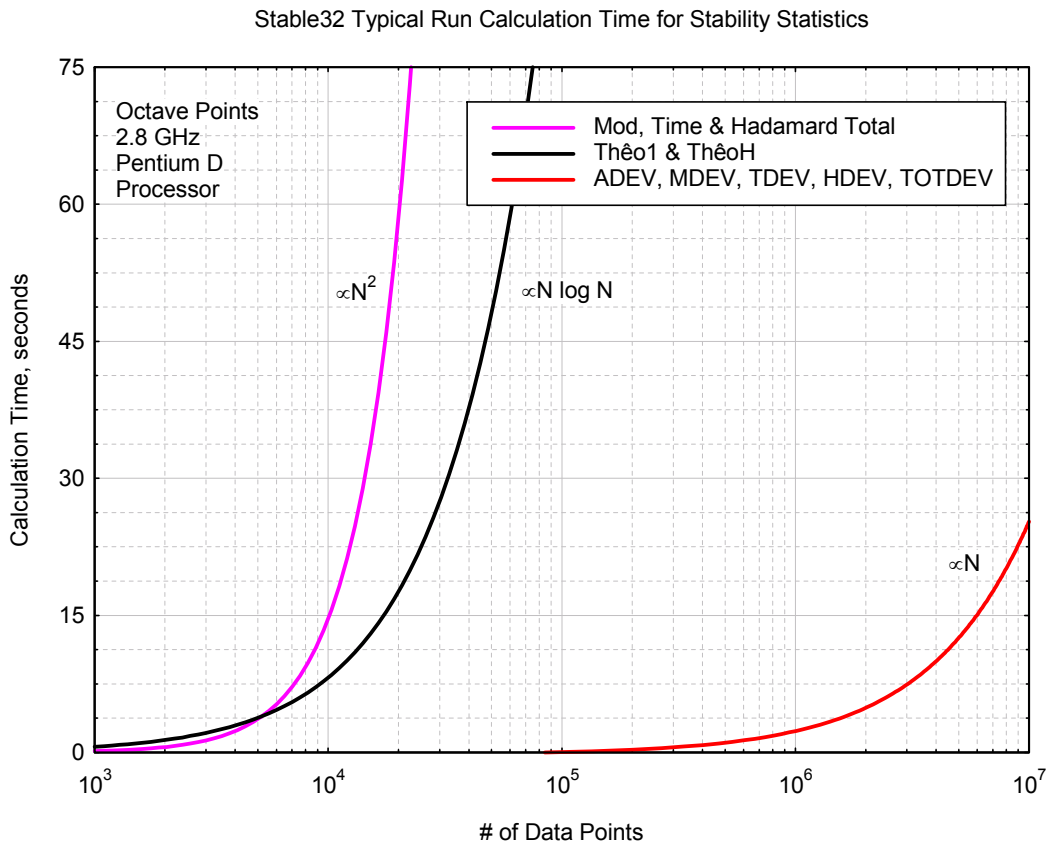
Stable32 supports the calculation of a number of statistics for frequency stability analysis, some of which can take a considerable time to compute, particularly for large data sets. This paper discusses some of the considerations that determine these calculation times, and strategies for managing them.

- **Basic Factors**

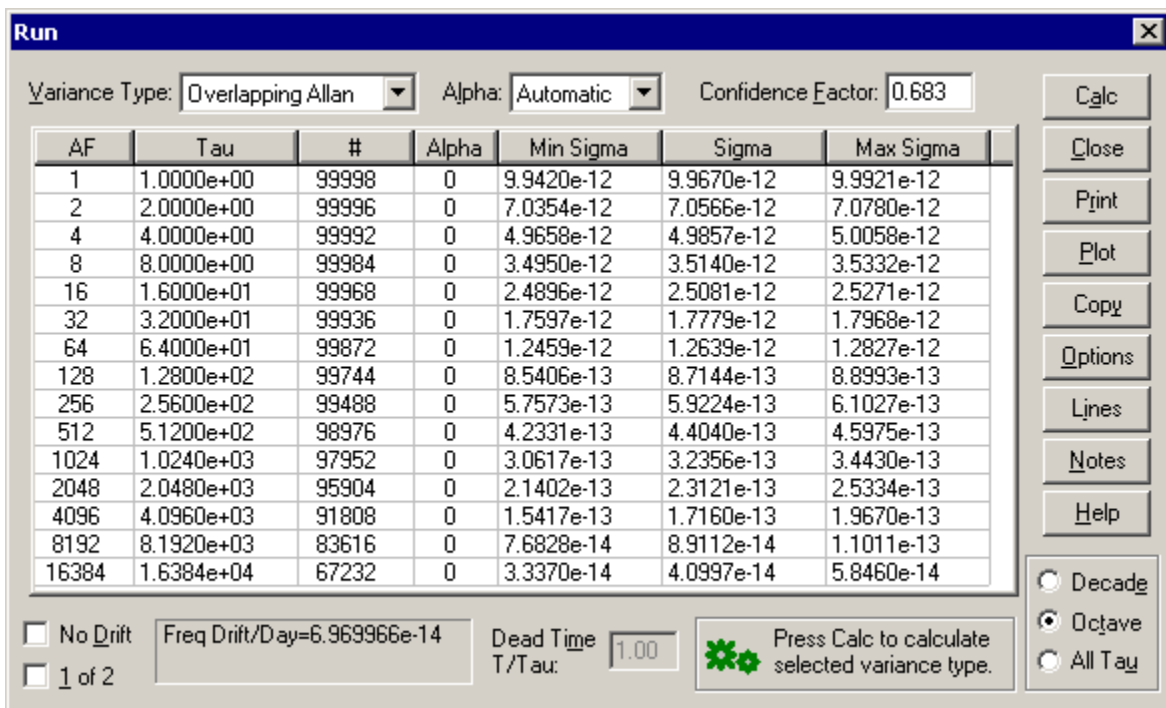
The basic considerations that determine the time required to perform a stability calculation are (1) the variance type, (2) the number of data points, and (3) the speed of the computer. The variance type, more particularly the algorithm implementing its estimation, determines the dependence of its calculation on the number of data points (e.g., proportional to N , $n \log N$ or N^2). The number of data points influences the calculation time accordingly, as obviously does the speed of the computer and its underlying numerical library functions. The analyst may, depending on the objectives of the analysis, have some control over the choice of variance type and the number of points (e.g., by data truncation or averaging). He might even invest in a faster computer (although that has seemingly reached somewhat of a plateau), and should also be aware of other choices and options available in Stable32 to reduce calculation time.

- **Variance Type Dependence on Data Set Size**

The most important factor in determining the calculation time dependence of a stability statistic on data set size is the variance type, as shown in the following plot.



This plot shows the basic calculation time dependence on the number of data points, N, of a Stable32 octave or decade run for four classes of common stability statistics. The simpler statistics of the Allan and Hadamard variance families (Allan, Overlapping Allan, Modified Allan, Time Allan, Hadamard, and Overlapping Hadamard) and the Total variance have a calculation time proportional to N. Each specific variance type has its own calculation time coefficient (only a single typical value is shown). Similarly, the Thêo1 and ThêoH statistics have a N log N dependence (log₁₀ is used herein), and the Modified, Time and Hadamard Total variance has a stronger N² dependence. The dependencies for the TIE rms and MTIE telecom stability statistics (not shown on the plot) are N and N² respectively. Again, those dependencies include an approximate log N term for a complete octave or decade spaced Stable32 analysis run for their specific range of averaging factors (both of which produce about log₂(10) = 3.3 points per decade of averaging time). For example, a typical Stable32 stability analysis run is shown in the following figure, in this case an octave-spaced overlapping Allan deviation run for a 100,000-point data set that required less than 1 second to compute.

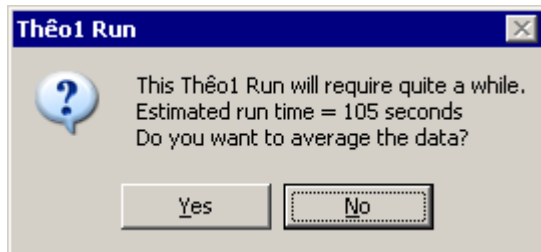


- **Calculation Time Coefficients**

Typical values for the various calculation time dependencies are shown in the following table.

Variance Type	Generic Dependence seconds	Typical Coefficient
Allan	$2.5e-6 \cdot N$	1.20e-06
Overlapping Allan		2.60e-06
Modified Allan		2.92e-06
Time		2.90e-06
Overlap & Mod Allan		4.16e-06
Hadamard		1.39e-06
Overlapping Hadamard		2.74e-06
Total		3.60e-06
TIE rms		2.30e-06
Modified Total		$1.5e-7 \cdot N^2$
Time Total	1.58e-7	
Hadamard Total	1.59e-07	
MTIE	$1.5e-8 \cdot N^2$	1.69e-08
Thêo1	$2.0e-4 \cdot N \log N$	2.45e-04
ThêoH		1.77e-04
Typical values shown for octave AF intervals with 2.8 GHz Pentium D processor		

These coefficients are automatically updated as described below. They are used to determine when a dialog box automatically opens (see below) to suggest averaging the data when the calculation time is likely to exceed a certain time (default = 60 seconds). This limit can be set by the Maximum Run Calculation Time item on the Run page of the Stable32 Configuration property sheet (see below). If set to 0, this feature is disabled.



Answering Yes to this dialog opens the Average function and then returns to the Run function. It also provides an easy way to abort the calculation.

Dialog Offering to Average Large Data Set



Configure/Run Dialog to Set Max Run Calc Time

- **Adaptive Calculation Time Estimation**

The Run calculation time coefficients are adaptive. They are automatically updated using 5-stage recursive averaging each time a long variance calculation is made, and the resulting values are stored in the [Run] section of the Stable32.ini configuration file. In that way, the particular Stable32 installation learns appropriate coefficient values to best estimate the run time for a particular variance type and data set size. The actual calculation time is also shown in the Stable32 status bar at the conclusion of each run.

- **Maximum Averaging Factor Determination**

Stable32 octave and decade stability runs use a maximum averaging factor determined by the number of data points and the specific variance type. Each variance type is assigned a stop ratio as shown in the following table, and the maximum averaging factor for a given stability run is determined by the number of data points divided by this stop ratio. The number of stability points produced by the run is approximately $\log_2(\# \text{ Data Points}/\text{Stop Ratio})$.

Variance Type	Stop Ratio
Allan	5
Overlapping Allan	4
Modified Allan	4
Time	4
Overlap & Mod Allan	4
Hadamard	5
Overlapping Hadamard	4
Total	2
TIE rms	4
Hadamard Total	3
MTIE	2
Thêo1	1
ThêoH	1
Modified Total	2
Time Total	2

- **Many Tau Averaging Factors**

Stable32 All Tau stability runs can use a “Many Tau” method to perform its calculations more efficiently by skipping unnecessary averaging factors, producing a stability plot with approximately equally-spaced points on a log tau scale. A user-entered Many Tau Size option on the Options page of the Stable32 Configure property sheet (default = 500) sets the number of points on the stability plot. During the run, a corresponding number of points per decade is determined and used to adjust the averaging factor increment to produce the desired point spacing. For an increment in the averaging factor of $x(i+1) = x(i) + x(i)/n$, the resulting averaging factor is a power function equal to $f(x) = (1+1/n)^x$ which produces $1/(\log_{10}(1+1/n))$ points/decade.

The calculation time of a many tau run can be estimated by the same method as used for an octave or decade run based on the ratio of the number of stability points. The octave/decade calculation time estimate would be multiplied by the Many Tau Size and divided by $\log_2(\# \text{ Data Points}/\text{Stop Ratio})$.

- **Thêo1 and ThêoH Bias Calculation**

The Thêo1 and ThêoH bias calculation takes about half the time in the determining those statistics, and does not have to be redone unnecessarily to repeat that stability calculation (e.g., for a different tau spacing). A relatively new and much faster ThêoH bias removal method is now used.

- **MTIE Calculation**

The fast MTIE calculation option takes about one-fifth of the time required by the normal octave or decade method at the expense of producing less regular tau values.

- **Other Stable32 Functions**

Execution time can also be an important consideration for other Stable32 functions. The DAVAR function performs Allan or Hadamard deviation calculations for multiple overlapping windows and presents those results in the form of a 3D stability plot. These calculations can involve a relatively large number of relatively large windows and therefore require a significant time. The subsequent interpolation and 3D plot meshing add about 20% more time. Only the latter has to be repeated if the plot orientation or other details are changed. A DAVAR plot with the default settings for a 1 million point data set takes about 2 minutes. The Power, Autocorrelation and Noise functions require about 100, 60 and 90 seconds respectively for 10 million data points, and the Statistics and status plots about 20 seconds each (the latter can be disabled in Configure/General). It takes about 70 seconds to open a 10 million point data file (exclusive of the Data File Opened plot, which can also be disabled).

- **Calculation Time Summary**

Typical Stable32 Run function calculation times are shown in the following table for the various variance types. The calculations shown in orange are impractical and those in red are effectively impossible without averaging.

Variance Type	# Data Points	Typical Calculation Time			
		10 k	100 k	1 M	10 M
Allan		< 1 sec	< 1 sec	1 sec	10 sec
Overlapping Allan				2 sec	25 sec
Modified Allan				2 sec	25 sec
Time					
Overlap & Mod Allan				3 sec	40 sec
Hadamard				1 sec	15 sec
Overlapping Hadamard				2 sec	30 sec
Total				3 sec	40 sec
TIE rms				2 sec	25 sec
Modified Total				15 sec	25 min
Time Total					
Hadamard Total					
MTIE		2 sec	150 sec	250 min	400 hrs
Thêo1		12 sec	100 sec	20 min	250 min
ThêoH					